Two-Part Tax Controls for Forest Density and Rotation Time

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005

[*] The views expressed herein are my own and do not reflect the views of the Food and Drug Administration or the U.S. Department of Health and Human Services.

[†] Thanks to Pete Wilcoxen, Don Fullerton and David Kendrick for their comments and ideas. Any remaining errors are mine.

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Many forest amenities are derived not only from the age of the trees, but also from the density of the trees. When an externality such as erosion control is considered, clear-cutting results in much larger damages than occur with selective cutting. This paper extends current methodology, allowing firms to optimize over both rotation time and the commercial use percentage per acre. A two-part instrument, a “clear-cut” tax combined with a lump sum “licensing fee”, controls for commercial use percentage and rotation time in a firm that does not internalize non-timber benefits. Optimal taxes are presented that correct the firm’s suboptimal behavior. A two-part instrument is shown to remedy market failure when a private firm clear-cuts and harvests too soon, and when an overgrown forest is not privately optimal to maintain. Numerical analysis on a simple case, simulating a forest where the externality is erosion control shows the clear-cut tax policy’s relevance for a variety of erosion severity scenarios and interest rates.

Key Words:

Two-Part Tax

Forestry

Clear-cutting

Optimal Rotation

Externalities

Optimal Harvest

1. INTRODUCTION

This paper extends current literature of public policy on forestry by allowing the forest planner and private landowner to choose not only the rotation time, but also what portion of each acre will be used commercially.[1] Many forest amenities are derived not only from the age of the trees, but also from the density of the trees. In the case of some forest externalities such as erosion control, it is possible that the clear-cutting of a forest might result in much larger damages than would occur with selective cutting.

The regulation of the commercial use percentage per acre may dampen the externalities associated with clear-cutting and thus avert such disasters as the one experienced in Portland, Oregon in 1995. Massive landslides, occurring mostly in areas of clear-cutting, contaminated the entire city's water supply and caused millions of dollars of damage. The damage was so great that in a recent election, Oregonians were asked to vote on a referendum that would have banned clear-cutting completely. Regulating the commercial use percentage per acre may also dampen the externalities associated with letting a forest grow unchecked. The immense, costly forest fires occurring all over the Western U.S. in 2002 make it obvious that letting forests grow without management can be as bad as clear-cutting. Therefore, on forestlands that are in danger of falling victim to a wildfire and would not otherwise be logged, it may be prudent to provide a firm with incentive to harvest some percentage significantly greater than zero. In either case, the consideration of commercial use percentage is an important part of forest management and has been ignored by the forest economics literature thus far.

This paper focuses on two cases, one where a logging firm does not internalize the benefits from leaving some trees uncut and therefore chooses to clear-cut and the second where the forest in question is unprofitable to harvest and has become over grown. A pair of taxes is presented that, used in combination with one another, optimally control for clear-cutting rather than banning it completely. Furthermore, the same instruments can be used to induce the optimal behavior when it is socially optimal to harvest a forest, to prevent forest fires for example, but it is not privately optimal.

Section 2 of this paper briefly revisits the basic Faustmann and Hartman models of optimal forest management, then introduces a new model that allows a social planner to choose both the optimal rotation period and the optimal amount of each acre to use commercially. Section 3 sets up the private landowner's problem and includes taxes aimed at controlling both rotation time and percentage commercial use. Solutions are presented for the social and private problems, as well as the solutions for the optimal tax instruments. Section 4 shows the results of a numerical analysis on a simple stylized case.

2. FAUSTMANN, HARTMAN, AND BEYOND

Before jumping into a more general model, this section presents a brief overview of the classic Faustmann and Hartman models.

The Faustmann Model

The Faustmann model solves for the optimal rotation period of a forest that will be harvested

[1] Commercial use percentage, as will be discussed in detail later in the paper, means the amount of each acre a firm will choose strictly for its commercial value. A firm will choose this portion of each acre ex ante for all periods.

forever. Initially, the land is bare with timber production as its only use. The landowner chooses the rotation period T to maximize the following private value function:

$$V(T) = \frac{G(T)e^{-rT}}{1-e^{-rT}} \qquad (1)$$

where $G(T)$ is the net timber value of the stand at time T. The numerator is the present net value of the stand that will be harvested at T. The denominator is simply the result of summing over an infinite series of identical rotations.

Taking the derivative with respect to the rotation length and rearranging, the following first order condition is derived:

$$\frac{\dot{G}(T)}{G(T)} = \frac{r}{1-e^{-rT}} \qquad (2)$$

where the left hand side of the equation is the growth rate of the forest in terms of net value. So a rational landowner will cut down all the trees and replant when the growth rate equals the interest rate multiplied by a discount factor.

Hartman: Forest Planner Problem, Externalities Included

Following Hartman (1976), an externalities function that varies with the age of the forest is introduced into the forest planner's problem. The social value (*SV*) function can now be written

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The trees that are not part of the commercial use portion of each acre will never be harvested.

the following way:

$$SV(T) = \frac{\int_0^T e^{-rt} F(t)dt + G(T)e^{-rT}}{1 - e^{-rT}} \tag{3}$$

where *G(T)* is again the net timber value at age T, r is the interest rate, and *F(t)* is a function giving the externalities generated by the forest at each point in time. The integral gives the present value of a stream of non-timber benefits coming from the standing forest. Thus *SV(T)* describes society's total benefit from the forest, summed over an infinite series of rotations of length *T*.

The time path of the benefits flowing from a standing forest is controversial. Hartman (1976), Strang (1983), Snyder and Bhattacharyya (1990), Max and Lehman (1988),and Reed (1984), assume *F(t)* increases with the age of the trees at a decreasing rate. If this is the case, the externality function is monotonically increasing, and it is always optimal, disregarding the timber value, to let the forest grow. Even though this case may be the most likely, it is not the most general case. Calish et. al. (1978) show that, depending on the forest, the externality function could take any number of shapes, including a non-monotonic one. Englin and Klan (1990) assume only that *F(t)* is either increasing at a decreasing rate or increasing and then, at some point, decreasing. In this case, the externality function, at some time t_{Fmax}, begins to decrease and it is optimal (again disregarding timber value) to harvest the forest at some time less than infinity. Therefore, conceptually, the externality may "favor" either older trees or younger trees. This paper follows Englin and Klan's model in this regard.

Maximizing equation (3) with respect to the rotation length and rearranging, the first order condition can be written as: [2]

$$\frac{\dot{G}(T)}{G(T)} = \frac{r}{1-e^{-rT}}\left(1+\frac{\int_0^T [F(t)-F(T)]e^{-rt}\,dt}{G(T)}\right) \tag{4}$$

The social planner will allow cutting when the growth rate of the forest in net timber value is equal to a discount factor plus the "externalities balance", as dubbed by Englin and Klan (1990), divided by $G(T)$, the timber value at T. [3] Think of the externalities balance as the present value of the difference between the forest benefits available at the end of the rotation period, $F(T)$, and the stream of benefits available from the growing forest throughout the rotation period. Whether the optimal rotation time is shorter or longer than it would be without externalities depends on the sign of the second term on the right hand side. Furthermore, the sign of this term depends on the time path of the externality function. The externality, or amenity, favors old trees for example, when there is a positive difference between the amenity value at T and the amenity value at t, for all t, summed over the rotation period. This paper examines two cases. In the first case, it is assumed that for all $t < T$, $F(T) > F(t)$. This implies that consideration of the externality increases the optimal rotation time. This would be the case if the amenity were erosions control. In the second case, analogous to a fire prevention amenity, $F(t)>F(T)$ for some or all t, such that the sum of the differences, the externalities balance, over the optimal rotation period is negative. This implies that consideration of the amenity decreases the optimal rotation time.

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[2] The second order condition can be written as: $\dot{F}(T)+\ddot{G}(T) < r\,\dot{G}(T)$

[3] Note, at this point, as in previous literature, clear-cutting is assumed.

The private landowner who does not internalize the externality will not choose the socially optimal rotation period. Thus the need for corrective regulation or tax instruments arises.

Allowing for Choice in Percent Commercial Use and Rotation Timing

To capture the effects of the density of trees left standing, as well as the rotation period on forest amenities, two major features are added to the previous model. First the forest planner or the private landowner chooses not only when to cut, but also what percentage per acre to use commercially. This additional choice is important, since many forest amenities such as erosion control are directly related to the remaining density of the forest. It may not be optimal to let the external benefit reach zero, as might be the case with a clear-cut. This is especially true when the positive externality is control of a potential disaster, such as a flood, massive erosion, or fire. The firm in this case, before harvesting the first rotation, chooses once and for all what percentage of each acre of forest to use commercial harvest.[4] The remaining trees, the ones not within the commercial portion of each acre, are "tagged" to remain uncut for as long as they live.[5,6] Second, the externality function and the private gross revenues and costs, the timber value of the forest, are allowed to be functions of the percentage commercial use, as well as the forest's age.

The forest planner again maximizes the sum of the timber and non-timber benefits of the

[4] Interesting papers have been written addressing optimal forest thinning (Cawrse, et al (1984), Betters et. al. (1991)). The choice of commercial use percentage is not identical to the thinning choice, however. Because we are concerned primarily with externalities associated with clear cuts, this paper focuses on controlling the timing and the number of trees cut during the *final* harvest decision. For simplicity and to center attention on the final harvesting decision, within rotation thinning decisions on the commercial portion of the forest are left out of the model.
[5] The trees may be left in different patterns depending on the externality in question. For instance, if the externality is silt flow into a salmon habitat, a band of trees never-cut lining separating the stream and the commercially harvested portion of the land would be desirable. If the externality is erosion on the side of a hill, fire control, or viewing pleasure, the trees left uncut might be spread evenly throughout each acre.
[6] This restriction, that trees that are not harvested in the first period are left uncut once and for all, implies that the commercial use portion of the forest can be modeled as an even age forest.

forest over an infinite number of identical rotations. The value function can be written as follows:

$$SV(T,H) = \left(\frac{1}{1-e^{-rT}}\right)\left(\int_0^T e^{-rt} F(t,P_C)dt + G(T,P_C)e^{-rT}\right) \quad (5)$$

Note the difference between this expression and equation (4). Now the forest externality is both a function of age t and the commercial use percentage per acre P_C.[7] With respect to P_C, the path of $F(t, P_C)$, either decreases monotonically with the commercial percentage per acre, as would be the case if the amenity were erosion control, or follows an inverted “U’ shape, as would be the case if the amenity were fire control. The net timber value, $G(T, P_C)$, is an increasing function of both T and P_C. Since the forest planner must now make two choices, the optimization problem produces two first order conditions: one describing the optimal rotation time and the other describing the optimal percentage of commercial use per acre. These first order conditions are:

$$\frac{1}{G(T,P_C)}\frac{\partial G(T,P_C)}{\partial T} = \frac{r}{1-e^{-rT}}\left(1+\frac{\int_0^T (F(t,P_C)-F(T,P_C))e^{-rt}dt}{G(T,P_C)}\right) \quad (6)$$

$$\frac{\partial G(T,P_C)}{\partial P_C}e^{-rT} = -\int_0^T \frac{\partial F(t,P_C)}{\partial P_C}e^{-rt}dt \quad (7)$$

Equation (6) gives the conditions for optimal rotation time given a certain commercial percentage per acre. The social planner will allow harvest when the growth rate of the forest in net timber value is equal to a discount factor plus the externalities balance divided by the timber value at T and harvesting fraction P_C.

Equation (7) gives the first order condition for the optimal percentage of commercial use per acre given a certain rotation period. The social planner will choose P_C such that the marginal net timber value with respect to P_C is equal to the marginal value of the lost forest amenity with respect to P_C. Intuitively, the social benefits of a marginal increase in P_C, derived from an increased amount of timber harvested, must equal the social costs of a marginal increase in P_C, derived from a decrease in the amenity value of the standing forest. By definition, the timber value of the forest is increasing in P_C. Therefore, the social planner must choose a percentage commercial use such that the amenity value of the forest is marginally decreasing. An internal solution to the problem requires that $F(t, P_C)$ be decreasing in P_C, but monotonicity is not necessary. Nonmonotonicity in P_C may occur if the amenity is fire prevention.

3. PRIVATE OPTIMIZATION, TAXATION, AND SOCIALLY OPTIMAL OUTCOMES

This section examines the private logging firm's problem, where the firm faces taxes aimed at controlling both the percentage of commercial use on each acre and rotation time. If feasible to implement, a Pigovian tax or subsidy on the externality would directly force the firm to internalize the externality and could induce optimal behavior in both choices. However, a Pigovian tax or subsidy is not feasible in this case. If the externality in question were erosion control for example, a tax on the units of dirt that travel from one acre to the next, or into a

[7] By definition, $0 \geq P_C \geq 1$. $P_C = 1$ would be a clear-cut.

stream, may be unrealistic. The observation of such an externality and enforcement of the tax would be very difficult, if not impossible.

Since the firm can choose both the rotation period and the commercial use percentage, and a Pigovian subsidy is unrealistic, a single instrument cannot control both choices. An alternative is to tax the market transactions and suboptimal behavior of which the externality is a symptom. This section analyzes the case where the clear-cut tax, combined with the licensing fee can correct the market failure in two general cases: one where the logging firm clear-cuts and harvests sooner than socially optimal, and one where the it is socially optimal to harvest a forest, to prevent forest fires for example, but it is not privately optimal. Solutions for the optimal commercial use percentage and rotation time are presented, then analyzed through comparative statics to characterize the effects of the two-part tax. Finally, this section solves for the optimal rates of tax and analyzes the proper application of the tax instrument to forests with different growth and externality characteristics.

Consider two tax instruments: a flat, lump sum licensing fee (τ^{LS}), collected at harvest time[8] and a "clear-cut" tax (τ^{CC}), levied on the firm's fraction of commercial use per acre.[9] The number of trees that can be planted on an acre is assumed to be fixed.[10] Although not a tax on a direct market transaction, and therefore having some observation and enforcement issues, the clear-cut tax is simpler to implement than the Pigovian tax. Enforcing the tax will require a

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[8] To control for rotation period alone, Englin and Klan (1990) suggest the use of existing tax instruments such as a property tax, a severance tax, or a yield tax. While these existing, market based instruments are interesting, intuitive tools that can be used to correct for a sub-optimal rotation period, they do not work in combination with the clear-cut tax to correct for both commercial use percentage and rotation time.

[9] τ^{CC} is different from a yield tax levied on the harvest revenue, or a unit tax levied on the timber volume of the harvest, as discussed in (Englin and Klan (1990), Koskela and Ollikainen, (2001), (2003)), as it is a tax on the *percentage* of each forested land unit allocated by the land owner as commercially harvested. Though a graduated per acre unit tax would work in a similar fashion, the "clear-cut" tax allows the forest planner a flexible way to more directly address the issue of post harvest forest density.

record from each site of what percentage of each acre the firm leaves uncut and assurance that trees chosen for their amenity value, that is the trees not part of the commercial use portion of the land, remain uncut.

Private Optimization (All Taxes Included)

For clarity of presentation, it is useful to define some functions:

$$b(t) = n(t)g(t) \tag{8}$$

where $n(t)$ is the number of trees, $g(t)$ is the timber volume of a tree at any given time. Thus b(t) can be interpreted as the timber volume of the growing trees in the stand at any given time. The timber volume is assumed to be monotonically increasing for all t. Although the number of trees will decrease over time, each tree grows fast enough keep the volume of timber increasing.

To implement the taxes properly, the timber value of the forest, $G(T,P_C)$, must be more clearly defined. Let it be broken down into net revenues and costs so that $G(T,P_C) = pP_C\, b(T) - c$, where p is the price of timber, net of unitary harvest costs, and c is the fixed harvest and replant cost, a weakly increasing function of P_C.[11] The large lump sum costs of harvest that a firm must pay each period for capital and labor are captured by c. The intuition is that firms must rent a certain amount of equipment and labor to harvest each acre regardless of the commercial use percentage. Incidental marginal costs are captured in p. The term $pP_C b(T)$

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[10] The policy maker would have to consider the species of tree being planted. A different number of trees could be planted per acre, depending on the species. Furthermore, different forest types will have a different optimal percentage of uncut trees.

[11] This method to define $G(T, P_C)$ loosely follows Englin and Klan (1990).

can be interpreted as the value of the growing trees at time T. Now it is possible to define the firm's profit function, net of taxes, at the time of harvest:

$$\pi(T, P_C) = pP_C b(T) - c - \tau^{LS} - \tau^{CC}\theta(P_C) \tag{9}$$

The actual penalty incurred from the clear-cut tax is the tax rate, τ^{CC}, multiplied by $\theta(P_C)$, an increasing, convex function of P_C.[12] This allows the instrument to take the form of a continuously graduated tax if desired by the planner.

The private landowner chooses T and P_C to maximize the following value function:

$$PV(T, P_C) = \frac{1}{1-e^{-rT}}\pi(T, P_C)e^{-rT} \tag{10}$$

Notice that the private firm does not internalize the externality. The left-hand side of the equation simply reflects the fact that the private value function depends on T and P_C.

Maximizing over the rotation period and percentage commercial use yields the following first order conditions:

$$\frac{\partial \pi(T, P_C)}{\partial T} = \left(\frac{r}{1-e^{-rT}}\right)\pi(T, P_C) \tag{11}$$

[12] Alternatively, τ^{CC} could be multiplied by $\Gamma(P_C)$, a decreasing, concave function of P_C. If this were the case, τ^{CC} could take the form of a subsidy. The subsidy would decrease in P_C and could entice a firm choose the optimal percentage commercial use per acre. For the purposes of this paper, I analyze only the most intuitive case where τ^{CC} is an increasing penalty on P_C.

$$\frac{d\pi(T, P_C)}{dP_C} = 0 \tag{12}$$

Equation (11) shows the firm will harvest when the benefit of a marginal increase in T equals the discounted net profit gained by harvesting at time T. If $pP_C b(T) < c$, the firm will never harvest unless subsidized. Equation (12) shows the firm will choose P_C such that the net gains in increasing the commercial use percentage equals zero. With fixed costs and no taxes the firm will arrive at a corner solution, where $P_C = 1$.

Plugging the profit function and the proper partial derivates into 11 and 12 yields:

$$pP_C \frac{db(T)}{dT} = \left(\frac{r}{1-e^{-rT}}\right)(pP_C b(T) - c - \tau^{LS} - \tau^{CC}\theta(P_C)) \tag{13}$$

$$pb(T) = \tau^{CC} \frac{d\theta(P_C)}{dP_C} \tag{14}$$

The intuition for equations (13) and (14) are similar to the previous two equations, though it is clear now, from equation (14) that with constant costs and no tax, an internal solution for P_C does not exist.

The Growth Function, Externalities, and the Effect of Clear-cutting: Two Cases

Before launching into analysis of the implementation of clear-cut tax, lump sum licensing fee combination, it will be helpful again to examine the possible forest characteristics that might influence the optimal tax combinations as well as the magnitude of their actual effect..

Case 1: Before the planner arrives, the private landowner is clear-cutting and harvesting the trees sooner each period than is optimal. No prior restrictions are made on the magnitude of the effects of P_C and T on the externality, only on the sign. In this case, $F(t,P_C)$ is strictly increasing over T, for all $t<T$, and strictly decreasing in P_C. P_C may have a large effect on the externality in question, as it would be in the case of erosion control on a steep slope, or in the preservation of a species habitat. Or the effect may be a small one, as might be the case if the forest is used for hiking. Furthermore, no assumption is made as to the rate of growth of the forest.

Case 2: The forest in question is unprofitable to harvest and has become over grown. In this case, the forest planner must induce harvest to decrease an externality, such as fire risk, associated with *too many* trees on each acre. Positive P_C now acts a fire inhibitor. The externality thus takes an inverse U shape in P_C. Over a relatively low commercial percentage per acre chosen, the amenity is increasing. More trees harvested lead to greater fire prevention. However, over a relatively high P_C the amenity is decreasing. So the forest planner wants to induce some harvest, but a clear-cut still does not maximize the amenity value. This would be the fairly general case under decreasing marginal amenity returns or where there are multiple amenities, such as erosion control, scenery, and existence value, besides fire control, that are increasing in P_C.[13] Further, the externality favors younger trees, that is $F(t,P_C)$ is decreasing for some or all t such that the externalities balance is positive. As in the first case no prior restrictions are made on the magnitude of the effects of P_C and T, or the growth rate of the forest.

Thus, the following section shows how the two-part instrument can be used to increase or decrease a firm's commercial use percentage and rotation period under fairly general conditions.

Two Part Instrument: Clear-cut and Lump Sum Licensing Fee

Differentiating equation (13) with respect to τ^{LS} yields the effect of the yield tax on the optimal private rotation period:

$$\frac{dT^*}{d\tau^{LS}} = \left(\frac{r}{1-e^{-rT}}\right) > 0 \tag{15}$$

Differentiating equation (13) with respect to τ^{CC} yields the effect of the clear-cut tax on T*:

$$\frac{dT^*}{d\tau^{CC}} = \left(\frac{r}{1-e^{-rT}}\right)\theta(P_C) > 0 \tag{16}$$

A positive lump sum tax lengthens the rotation period. With a lump sum licensing fee paid every rotation, the competitive private owner must let the trees mature longer in order to maximize profits. The effect of the clear-cut tax on T* is the present value of the marginal clear cut tax penalty. This is unambiguously positive. When a logging company is persuaded to harvest fewer trees than is privately optimal, it must let the trees grow longer to maximize profits. Since both taxes serve to lengthen the optimal rotation period, the effect of the two-part instrument, where both taxes are positive, is to unambiguously lengthen rotation time. It is also possible that a properly set subsidy tax instrument could be used to arrive at the optimal T.

[13] Of course if the threat of fire were severe enough to outweigh the marginal alternate amenity value of even the last tree, $F(t, P_C)$ would be monotonically increasing in P_C. In this case, the result is a corner solution. The forest planner subsidizes the firm to induce profits ≥ 0, and sets $\tau^{CC} = 0$, allowing a clear-cut.

Differentiating equation (14) with respect to τ^{LS} yields the effect of the lump sum licensing fee on P_C*:

$$\frac{dP_C^{\ *}}{d\tau^{LS}} = 0 \tag{17}$$

Differentiating equation (14) with respect to τ^{CC} yields the effect of the clear-cut tax on P_C*:

$$\frac{dP_C^{\ *}}{d\tau^{CC}} = -\frac{d\theta(P_C)}{dP_C} < 0 \tag{18}$$

The licensing fee has no effect on the optimal commercial use percentage. This is expected. The licensing fee penalizes the firm a fixed amount every rotation, but does not penalize it based on how much of each acre it allocates for harvest. Therefore the firm's profit maximizing commercial use percentage is not influenced by the lump-sum licensing fee.[14] The effect of the clear-cut tax depends upon the marginal penalty received for cutting P_C. Since by definition $\theta(P_C)$ is increasing in P_C, the tax decreases the optimal commercial use percentage. Further, the net effect of the two-tax instrument on P_C is negative when both taxes are positive. The policy will always serves to decreases the optimal density of the harvest.

Note that while the clear-cut tax affects both P_C and T, the licensing fee only affects T. This is both an interesting and useful result. A planner wanting to control for commercial harvest percentage can set τ^{CC} to arrive at the optimal P_C, then use τ^{LS} to fine tune the timing of each harvest. If the effect of P_C on the externality is sufficiently large to produce the need for a large clear-cut tax, inducing an inefficiently long rotation period, τ^{LS} can take the form of a subsidy to reduce the rotation period by the optimal amount.

Setting private first–order conditions equal to social first-order conditions to solve for the optimal tax rates yields:

$$\tau^{CC} = -\int_0^T \frac{\partial F(t,P_C)}{\partial P_C} e^{-rt} dt \left(\frac{d\theta(P_C)}{dP_C} e^{-rT} \right)^{-1} \tag{19}$$

$$\tau^{LS} = \int_0^T (F(t,P_C) - F(T,P_C))e^{-rt} - \tau^{CC}\theta(P_C) \tag{20}$$

These tax rates make the private first order conditions match the socially optimal first order conditions. From equation (7) we know the social value function is maximized where $F(t,P_C)$ is decreasing in P_C, whether $F(t,P_C)$ is monotonically decreasing in P_C as in Case 1, or inverse U shaped in P_C as in Case 2. Thus, since the tax penalty is constant or graduated by definition, the optimal clear cut tax rate is positive in either case. The optimal tax rate increases as the effect of P_C on the externality increases. For example, if the externality were erosion control, the clear-cut tax would necessarily be higher for firms harvesting on sloped landscapes, or above a

[14] This is true only when there is an interior solution for the maximization problem relative to rotation time. If the forest is unprofitable to the private firm, a lump sum subsidy could be used to induce harvest of P_C>0. However,

populated area or water source. Intuitively, the clear-cut tax rate decreases as the tax penalty becomes increasingly graduated.

The sign and magnitude of the lump sum licensing fee depends upon the growth rate of the value of the externality and τ^{CC}. The lump sum fee is positive if:

$$\int_0^T (F(t,P_C) - F(T,P_C))e^{-rt} > \tau^{CC}\theta(P_C) \tag{21}$$

or, plugging in the optimal τ^{CC}:

$$\int_0^T (F(t,P_C) - F(T,P_C))e^{-rt} > -\int_0^T \frac{\partial F(t,P_C)}{\partial P_C} e^{-rt} dt \left(\frac{d\theta(P_C)}{dP_C} e^{-rT} \right)^{-1} \theta(P_C) \tag{22}$$

In Case 1, $F(t,P_C)$ is strictly increasing over T, for all $t<T$, so the externalities balance is negative. Therefore the optimal licensing fee is negative and takes the form of a subsidy. If the value of the externality grows slowly throughout the rotation, the lump sum subsidy will be larger. Furthermore, if the social value of the forest is very sensitive to the density of the trees, requiring a large clear-cut tax, the planner will need to pay a larger subsidy. A large τ^{CC} may give the firm incentive to wait longer to harvest than is optimal. The proper lump sum subsidy corrects this inefficiency.

In Case 2, the amenity is fire control, so $F(t,P_C)$ is decreasing over all or some of $t<T$, such that the externalities balance is positive. Thus it seems possible that equations (21) and (22) could hold. However, since the planner is attempting to induce harvest in an otherwise

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once the firm decides to harvest, the lump sum fee cannot control P_C.

unprofitable forest, the lump sum transfer must be a subsidy. Though $F(t,P_C)$ is inverse U shaped in P_C, equation (7), as discussed above, shows that the forest planner must choose a percentage commercial use on the decreasing part of the amenity function. If the fire control amenity is more sensitive to the age of the forest, rather the density, the externalities balance will be relatively large, and $F(t,P_C)$ will begin decreasing in P_C at a relatively low percentage commercial use. In this case, the planner will offer the firm a large subsidy to induce shorter rotation period, but will also tax the percentage commercial use relatively heavily. This results in more never-cut trees, with less time between "maintenance" harvests. If the fire control amenity is more sensitive to the density of the forest, rather than the age, the externalities balance will be smaller, and $F(t,P_C)$ will begin decreasing in P_C at a relatively high percentage commercial use per acre. In this case, the planner will offer the firm a smaller subsidy to induce harvest, but will also tax the percentage commercial use less heavily. This will result in more trees per acre cleared during harvest, but with slightly longer rotations.

In summary, the clear-cut tax affects both commercial use percentage per acre and rotation timing. However, in all except a unique case, the tax causes the firm's rotation period to be too short or too long. Furthermore, the clear-cut tax cannot induce harvest when the forest is not privately profitable. Therefore the second part of the two-part instrument, the lump sum licensing subsidy, is needed to correct for the inefficient rotation period.

4. A SIMPLE NUMERICAL EXAMPLE

So far the model has been very general and has been applicable to a wide range of cases. A numerical model will illustrate what the optimal tax rates are under different conditions. This section describes one very simple, but intuitive stylized example where the clear-cut tax and lump sum tax are implemented.

In the numerical example, a specific form for the externality function, $F(P_C,T)$ is assumed. The function below shows the instantaneous value of the amenity for any P_C and t:

$$F(t, P_C) = \gamma(1 - P_C{}^2)b(\infty) + \gamma P_C{}^2 b(t) \tag{23}$$

where γ is the dollar value per volume unit of the amenity. The first group of terms on the right hand side of the equation can be interpreted as the amenity value of the trees that have been left uncut and have reached peak volume. The second group of terms is the amenity value of the growing trees that have been replanted since the previous harvest. Squaring the harvest fraction puts more weight on the value of the fully-grown trees. This approximates the case where the concern is soil erosion. As the commercial use portion of the forest regrows, the new trees will develop root systems and eventually aid in the reduction of erosion. However, since trees are typically harvested long before they are full grown, it is the uncut trees with fully developed root systems that will produce the most erosion control.

For simplicity, p is assumed to be net of all costs.

Under these assumptions, the first order conditions for a social optimum become:

$$\frac{1}{G(T)}\frac{\partial G(T, P_C)}{\partial T} = \frac{r}{1 - e^{-rT}} + \frac{1}{pb(T)}\frac{\gamma r P_C}{1 - e^{-rt}}\int_0^T (b(t) - b(T))e^{-rt}dt \tag{24}$$

$$P_C = \frac{pe^{-rT}}{2\gamma}\left(\int_0^T \left(\frac{b(\infty)-b(t)}{b(T)}\right)e^{-rt}\,dt\right)^{-1} \tag{25}$$

The first order condition for the optimal rotation period depends linearly on γ and P_C, and P_C depends inversely on γ. Plugging the condition for optimal percentage commercial use into equation (24) shows that the optimal rotation period does not depend on γ:

$$\frac{1}{G(T)}\frac{\partial G(T,P_C)}{\partial T} = \frac{r}{1-e^{-rT}} + \left(1+\frac{e^{-rT}}{2}\int_0^T (b(\infty)-b(t))e^{-rt}\,dt\right)^{-1}\int_0^T (b(t)-b(T))e^{-rt}\,dt \tag{26}$$

For any given P_C, γ increases the optimal rotation period. However, γ also decreases the optimal P_C. These effects cancel out exactly and the optimal T is left unaffected by γ. The tax on the commercial use fraction will not affect the firm's decision on rotation length.

From equation (25), it is apparent that an internal solution, $0 < P_C < 1$, will exist if:

$$\frac{pe^{-rT}}{2\gamma} \leq \int_0^T \left(\frac{b(\infty)-b(t)}{b(T)}\right)e^{-rt}\,dt \tag{27}$$

This is true for a sufficiently small p or sufficiently large γ. There exists an internal solution as long as the ratio of marginal timber value to marginal amenity value is sufficiently small.

The clear-cut tax is equal to the square of P_C: $\theta(P_C) = P_C^2$. This penalizes the firm relatively more for a commercial use percentage closer to 100%. This reflects the idea that the planner is more concerned with reducing the amount of clear-cutting and cares less about a marginal change in the number of trees cut if the firm harvests as low fraction.

The stylized function used for the biomass of trees in board feet, *b(T),* is based on data published by Richard McArdle (1961) for the Douglas Fir.[15] The estimated function shown is:

$$b(T) = \frac{25{,}000}{1 + \mathrm{e}^{(-0.2(\mathrm{T}-35))}} \tag{28}$$

The function is plotted in Figure I. The biomass of this particular stand peaks at 25,000 board feet per acre and reaches this peak at a stand age of 70 years.[16]

The price of a board foot is taken from Calish et. al. (1978) and transformed into year 2000 dollars. In the model $p = 2.03$.

The model is implemented in the Ox programming language developed by Doornick (1999). T and P_C are solved simultaneously based on the estimated parameters. When the real interest rate is assumed to be 3% and γ is assumed equal to \$0.02, the Faustmann optimal rotation period, not considering the externality, is 41.6 years.[17] If the externality is considered, the optimal rotation period is 42.4 years and the optimal commercial harvest per acre is 55%. The optimal clear-cut tax is \$37,444 per square fraction of each acre for a total penalty of \$11,327 per

[15] The MacArdle data counts the amount of wood available from root to tip of each tree. Although a good estimate of the wood available to a pulp and paper firm, it does not provide an estimate of the wood available to be used as lumber. Thus, using the MacArdle data as a reference, the "stylized" biomass estimation captures roughly the growth in mass of the wood available for lumber.

[16] Obviously, trees will continue to grow past 70 years, but this model assumes that the wood useable for lumber will reach near maximum volume by 70 years.

[17] This is a point of reference only. In reality, as in the rest of the analysis, these values will vary.

acre. The optimal lump sum payment per rotation period is a subsidy of $8,891. Before taxes, the firm makes a gross profit of $22,822 per acre. The firm makes an after tax net profit of $20,302 per acre.

As predicted by the general theoretical model, it is possible, if not likely, that the lump sum licensing fee will take the form of a subsidy in order to correct for the heavy clear-cut tax necessary to reach the optimal percentage commercial use per acre. In the case above, the firm is charged a clear-cut penalty that is nearly half of their gross margins on timber. Furthermore, the firm is harvesting half of the lumber it would harvest in the absence of the tax. Therefore, to reach the optimal rotation time and make up for the clear-cut penalty that induces the optimal harvest percentage but misses the optimal rotation time, the firm must be subsidized each rotation period.

To better illustrate this point, the model is run under the previous parameter settings with the tax rates held constant at incorrect values. The lump sum tax is set to zero and the clear-cut tax rate is held at what optimal in the previous example ($37,444 per acre). Under this tax schedule, the firm will choose a rotation period of 46 years and a harvest percentage of 61%. The firm lets the trees grow for four years longer than is socially optimal, and because of the longer rotation period, the firm harvests 11% more trees than is socially optimal in order to maximize net profits. Without the corrective lump sum subsidy, the logging firm will harvest too much, too late.

To evaluate the sensitivity of the model to the assumptions about r and γ, the model is run with a range of values for these parameters. The effects of changing the discount rate, holding γ constant, on the optimal rotation period and harvest percentage are shown in Figure II. As shown in the figure, the optimal commercial use percentage decreases fairly rapidly as the discount rate increases. If the discount rate is high, the present value of the amenities a tree generates (which begin flowing immediately) will outweigh the present value of harvesting the tree at the end of each period. The amenity is worth relatively more when the discount rate is higher. Although T is not affected by the externality, it is affected by the discount rate. It is apparent from Figure II that the higher the discount rate, the shorter the rotation period. If the firm's revenue stream is discounted at a higher rate, it will harvest sooner. The decision of P_C seems to be more sensitive to changes in the assumed discount rate than does the decision of the optimal rotation period. With a discount rate of 7%, the optimal rotation period is 37.7 years and the optimal commercial use percentage is 18%. Thus the optimal rotation period decreased less than 12% from the previous example, where r = 3%, but the optimal commercial use percentage decreases by 37%.

The effect of changing the marginal amenity value, γ, holding r constant, on the optimal rotation period and commercial use percentage are shown in Figure III. As predicted, the optimal rotation period is unaffected by the marginal value of the amenity. The optimal P_C, decreases as γ gets larger. As the units on x-axis double, the optimal P_C decreases by one half. There is a seemingly linear inverse relationship between commercial use percentage and marginal amenity value. At γ= \$2.56/board foot, the optimal becomes no harvest at all. This is analogous to the case where each tree cut might drastically increase the chance of a landslide or damaging erosion. And although not shown on the figure, values of γ near \$0.01 produce a corner solution , where it is best to clear-cut. The marginal amenity value of each tree is so low that it is better to chop all of the trees down to reap the timber value.

The different optimal tax rates for changing values of r and γ, are shown in Table I. As expected, the clear-cut tax rate increases as the interest rate or γ increases. However, the actual tax penalty, $\tau^{CC} * P_C^{\ 2}$, decreases. This is due to the graduated nature of the tax. Notice that in no case is the lump sum transfer positive. In every non-corner solution case under the assumptions of this particular model, the dual tax instrument will take the form of a clear-cut and a lump sum licensing subsidy. It is likely that a different set of assumptions on the functional form of the externality would yield different results.

5. CONCLUSIONS

The consideration of commercial use percentage introduces a formidable, interesting, and useful economic problem. A two-part instrument, a clear-cut tax coupled with a lump sum subsidy can be used to remedy inefficiency where a valuable forest amenity is being negatively affected by a firm that is harvesting too many trees too fast. Furthermore, the same instruments can be used to induce harvest when it is socially optimal to harvest a forest, but it is not privately optimal. The taxes would be fairly easy to implement and could be used in a wide variety of real situations.

The stylized numerical model shows the wide variety of conditions where the clear-cut tax and lump sum transfer could be used to achieve a socially optimal solution when the forest amenity is analogous to erosion control. In the future, this model could be refined and expanded upon in many ways. For example, it would be interesting to find numerical solutions for the social optimum under different amenity value functions. Furthermore, interesting work has been done recently by Uusivuori and Kuuluvainen (2005)on multiple age forests. Relaxing the never-cut constraint would make this policy tool even more flexible and useful.

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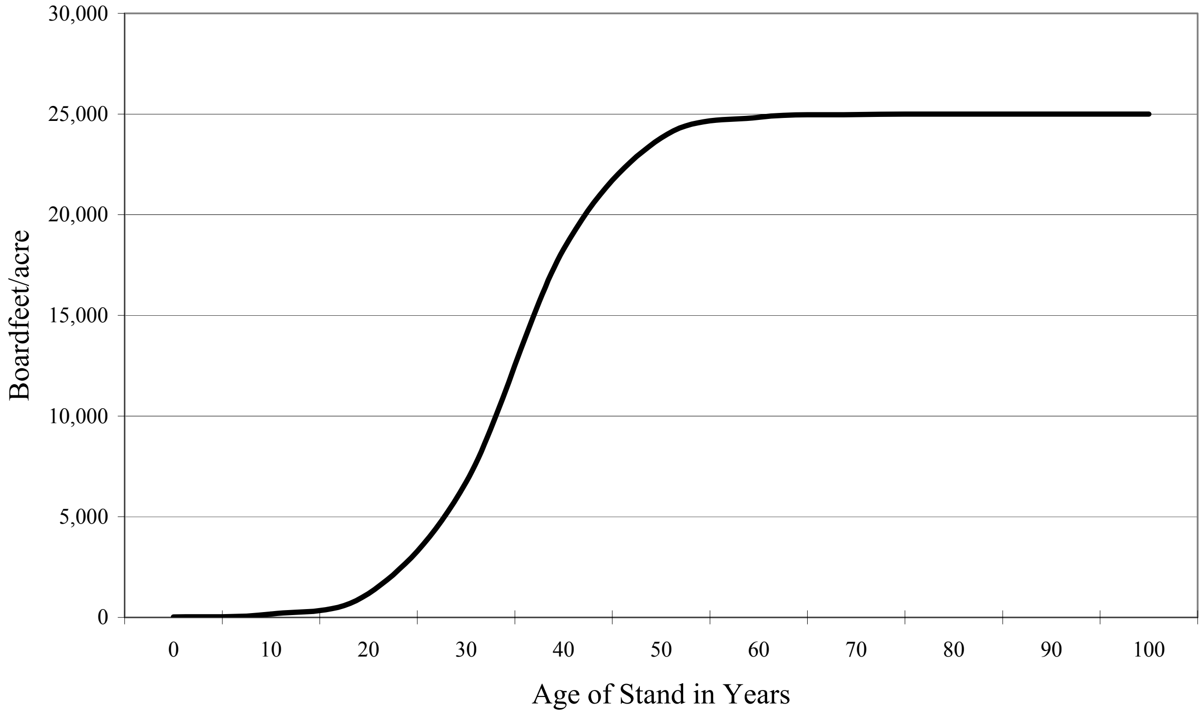
Figure I: Stylized Growth Function for Single Acre



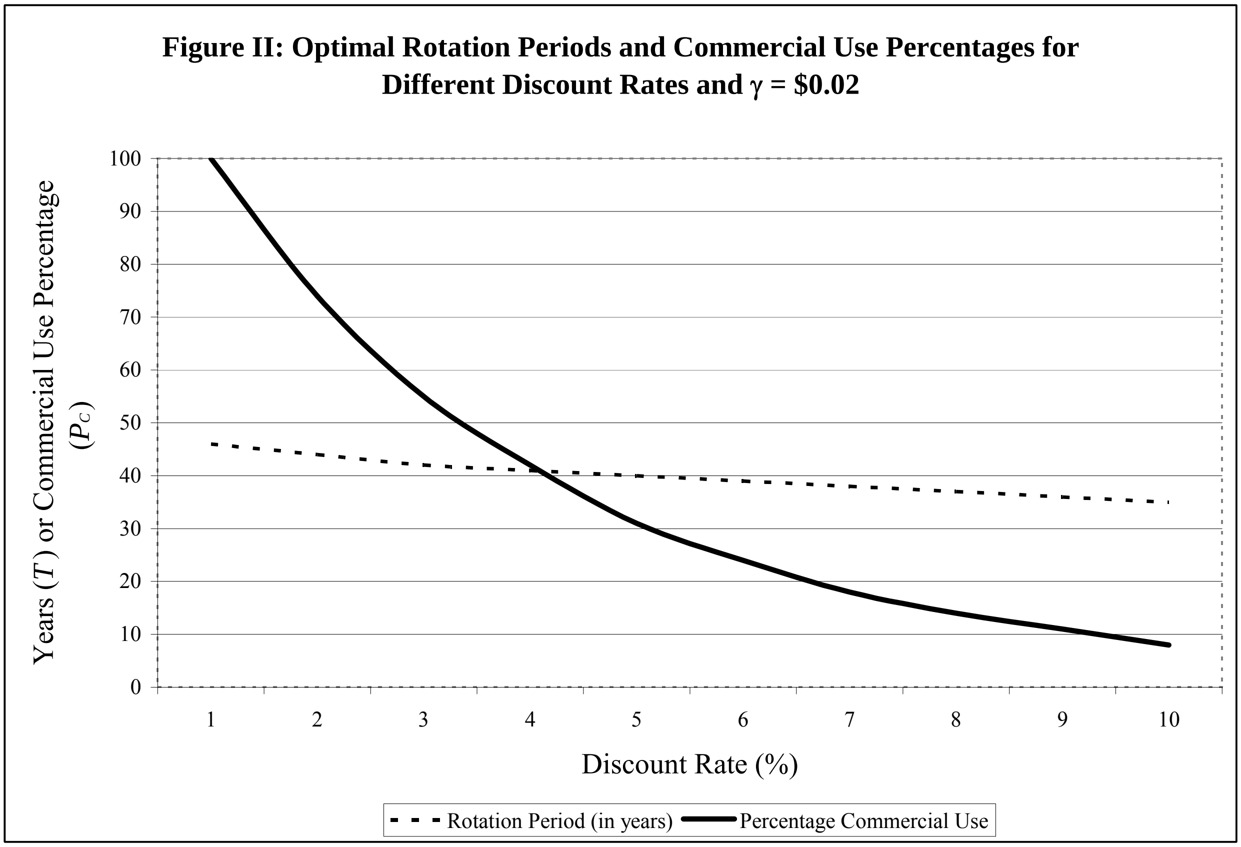


Figure II: Optimal Rotation Periods and Commercial Use Percentages for Different Discount Rates and γ = $0.02
Years (T) or Commercial Use Percentage (Pc)
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
Discount Rate (%)
Rotation Period (in years)
Percentage Commercial Use

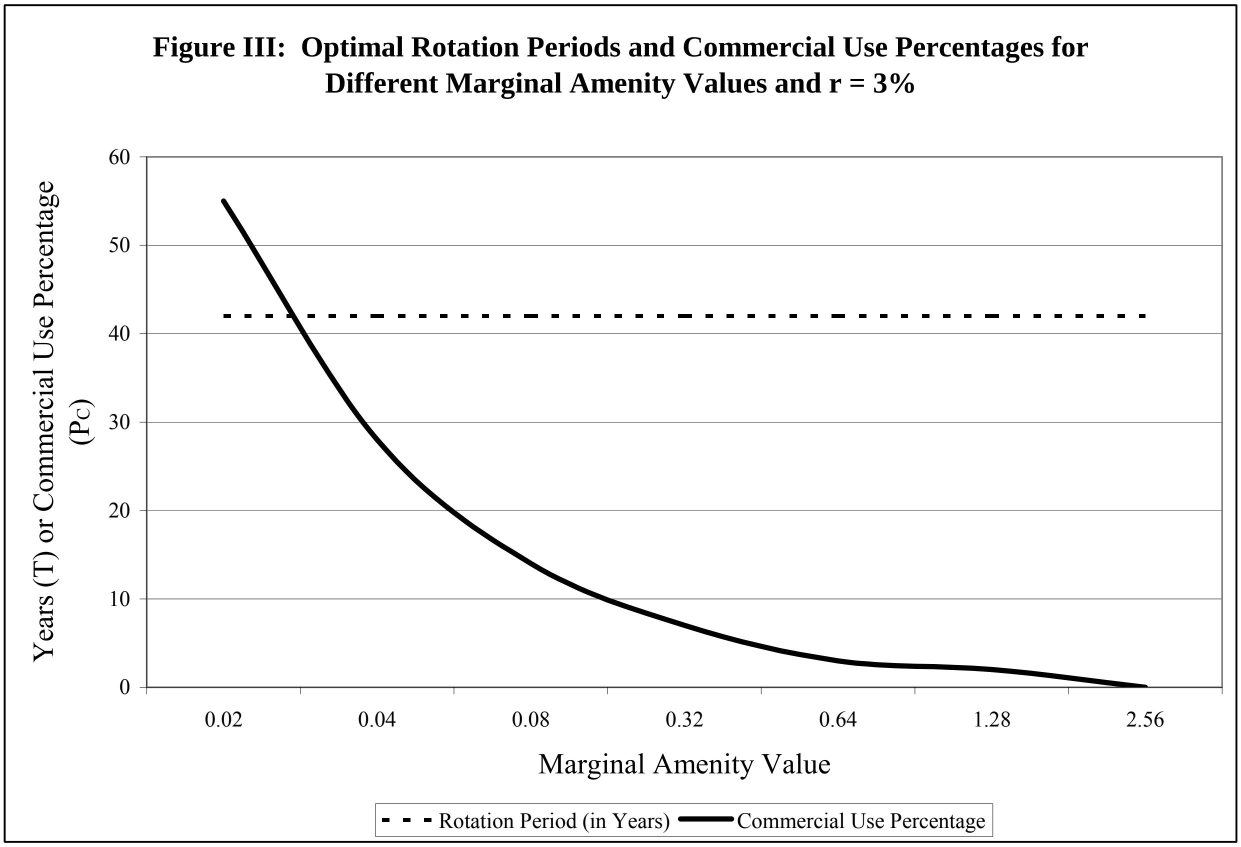


Figure III: Optimal Rotation Periods and Commercial Use Percentages for Different Marginal Amenity Values and r = 3%
Years (T) or Commercial Use Percentage (Pc)
60
50
40
30
20
10
0
0.02
0.04
0.08
0.32
0.64
1.28
2.56
Marginal Amenity Value
Rotation Period (in Years)
Commercial Use Percentage

Chart I: Effect of *r* and γ On Tax Rates and Penalties			
r	τ^{cc} (rate)	$\tau^{cc} * P_C^{\ 2}$ (penalty)	τ^{ls}
1%	$22,944	$22,944	-$10,234
2%	$29,522	$16,166	-$10,539
3%	$37,444	$11,327	-$8,891
4%	$47,014	$8,293	-$6,946
5%	$58,534	$5,625	-$5,219
6%	$72,292	$4,164	-$3,832
7%	$88,541	$2,869	-$2,771
8%	$107,449	$2,106	-$1,982
9%	$129,030	$1,561	-$1,404
10%	$153,050	$980	-$986
γ	τ^{cc} (rate)	$\tau^{cc} * P_C^{\ 2}$ (penalty)	τ^{ls}
0.02	$37,443.75	$11,326.73	-$8,890.93
0.04	$74,887.49	$5,871.18	-$4,445.47
0.08	$149,774.98	$2,935.59	-$2,222.73
0.32	$299,549.97	$1,467.79	-$1,111.37
0.64	$599,099.94	$539.19	-$555.68
1.28	$1,198,199.87	$479.28	-$277.84
2.56	$2,396,399.74	$0.00	-$138.92